INDIAN MARITIME UNIVERSITY

(A Central University Government of India)

END SEMESTER EXAMINATIONS-June/July 2019

B.Tech (Marine Engineering)

Semester-I

Mathematics-I (UG11T3102)

Date: 11-07-2019 **Maximum Marks: 100**

Duration: 3 hrs **Pass Marks: 50**

PART – A **(3 x10 = 30)**

Compulsory Questions: (The symbols have their usual meanings.)

1. (a) Find the nth derivative of $y = \frac{x}{(x-1)(2x+3)}$.

(b) If $z = e^{ax+by} f(ax - by)$ prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

(c) If $u = x^2 - y^2$, $v = 2xy$ and $x = r\,cos\theta$, $y = r\,sin\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$.

(d) Find the radius of curvature at any point $(at^2, 2at)$ of the curve $y^2 = 4ax$.

(e) Prove that $\Gamma(n+1) = n\Gamma(n)$.

(f) Evaluate the integral $\int_0^1 \int_0^y xye^{-x^2} dxdy$.

(g) Find the unit normal vector to the surface $xy^2z = 3x + z^2$ at the point $(-1, -1, 2)$.

(h) Using Cayley Hamilton theorem find the A^{-1} of matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

(i) Show that shortest distance between two points in a plane is a straight line.

(j) Graphically find the maximum value of $Z = 3x_1 + 2x_2$ subject to the constraints $3x_1 + x_2 \leq 15$, $x_1 + 2x_2 \leq 10$, $x_1,\ x_2 \geq 0$.

PART – B **(14 x 5 = 70)**

Answer any <u>FIVE</u> of the following questions

2 (a) If $cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$$ **[7]**

2(b) Find the asymptotes of the curve $y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0$. **[7]**

3(a) If $u = \tan^{-1}\frac{x^3+y^3}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ and

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u.$$ **[3+4]**

3(b) Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. **[7]**

4(a) Evaluate the double integral $\iint (x + y)dydx$ over the region bounded by $x = 0$, $x = 2$ $y = x$ and $y = x + 2$. **[7]**

4(b) Evaluate triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdz\, dy\, dx$. **[7]**

5(a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the component of the velocity and acceleration at $t = 1$ in the direction $i - 3j + 2k$. **[7]**

5(b) Show that the vector field $\vec{F} = (3x^2 + 3yz)\hat{\imath} + (3y^2 + 3xz)\hat{\jmath} + (3xy)\hat{k}$ is irrotational. Find a scalar potential function $\emptyset$ such that $\vec{F} = \nabla\emptyset$. **[7]**

6(a) Discuss the consistency of the following system of equations and solve it if consistent.

$x + 2y + z = 2$,
$3x + y - 2z = 1$,
$4x - 3y - z = 3$
$2x + 4y + 2z = 4$ **[7]**

6(b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. **[7]**

7(a) Let $F(z) = u(x, y) + iv(x, y)$ be an analytic function of z. If $u = x^4 - 6x^2y^2 + y^4$ then find v and express $f(z)$ in terms of z. **[7]**

7(b) Evaluate the integral $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = 3$. **[7]**

8(a) Find the curve on which functional $\int_0^2 (x + y')y'dx$ with $y(0) = 0$ and $y(2) = 1$ can be extremized. **[7]**

8(b) Using simplex method solve the following LPP

Maximize $Z = 5x_1 + 3x_2$
subject to $x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1, x_2 \geq 0$. **[7]**

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